On quasi-bound states in η- nucleus systems

J.Kulpa, S. Wycech*
Soltan Institute for Nuclear Studies, Warsaw, Poland

A.M. Green†
Department of Physics and Helsinki Institute of Physics, P.O. Box 9, FIN–00014 University of Helsinki, Finland

The optical potential for the η-meson is calculated. This is based on a new determination of a large η-N scattering length and recent experiments of η production in two nucleon collisions. These determine the η - nuclear potential well to be about 45 MeV deep and able to support S, P, D and F quasibound states. Some of the higher angular momentum states are fairly narrow and offer a chance of experimental detection.

PACS numbers: 13.75.-n, 25.80.-e, 25.40.Ve

*e-mail "wycech@fuw.edu.pl"
†e-mail "anthony.green@helsinki.fi"

I. INTRODUCTION

The possibility of η-nuclear quasi-bound states was first discussed by Haider and Liu [1] and Li et al. [2], when it was realised that the η-nucleon interaction is attractive. Nevertheless, the early (π,p) experiment looking for these effects was not conclusive [3]. One possible reason for the difficulty in the interpretation of those experimental results is that these states are very broad. This point of view has been presented in Ref. [4], where the widths of η nuclear states were attributed largely to the two nucleon capture mode. Another possibility, pursued in this letter, is that these states are bound much more strongly than was generally expected. If the quasi-bound states exist, then one may expect them to be narrow, in few-nucleon systems, and thus be easier to detect there. An indirect verification was suggested by Wilkin [5], who interpreted a rapid slope of the $pd \rightarrow \eta^3$He low energy amplitude as a signal of a quasibound state. More recently in a simpler two nucleon case, very strong three-body $pp\eta$ correlations were found in measurements of the $pp \rightarrow pp\eta$ cross section in the threshold region [6].

Today, it is possible to revive the question of η nuclear states on a firm experimental background. First, definite progress has been achieved in the understanding of low energy η-N interactions. This allows us to calculate the dominant single-nucleon optical potential for an η in nuclear matter. Second, the measurement of η production in the nucleon collisions allows us to calculate the absorptive part of the two-nucleon contribution to this optical potential.

One particular result of the recent studies is the scattering length $a_{\eta N}$ in the η-N system, which is expected to be much larger than was previously estimated. A recent value of $a_{\eta N} = 0.75 + i0.27 fm$ has been obtained in Refs. [7] and [8]. Also , several other recent models listed in Ref. [7] produce $\Re\, a_{\eta N}$ much larger than the $0.3 fm$ expected, when the first calculations of η nuclear binding were performed. These models combine interactions in the coupled η-N , π-N , γ-N , $\pi\pi$-N channels and use the scattering data to fix the properties of the η-N scattering amplitude in the threshold region. In particular a scattering amplitude as large as $0.75 fm$ would generate an η-nucleus potential as deep as -80 MeV and a very rich η-nuclear spectroscopy. Such an estimate is not realistic, however, and it will be shown below that off-shell and nuclear effects reduce this well depth to about -50 MeV. Even so, it is strong enough to bind S, P, D, F states in heavy nuclei. In addition an absorptive amplitude of $0.27 fm$ would generate an absorptive potential W_N as strong as 30 MeV in the nuclear center and the widths of S states as large as 50 MeV. Such short lived states would certainly be very difficult to detect. Again, the off-shell and nuclear effects reduce this absorptive potential strength to about 10 MeV.

In addition, experimental cross-sections for the $NN \rightarrow NN\eta$, and $d\eta$ reactions determine that part of the absorptive potential which describes the inverse $\eta NN \rightarrow NN$ reaction rate in nuclei. These reactions are essential to fix an uncertainty that has earlier existed in the estimates of the η lifetime in a nucleus. The two-nucleon capture processes are found to generate a rather weak absorptive potential W_{NN} of ≈ 3.2 MeV strength at nuclear matter densities. Furthermore, this mode of η absorption is found to be dominated by the spin triplet NN pairs. Altogether, the optical potential calculated here supports rather a rich nuclear spectroscopy for the η mesons. In particular, many 20 MeV broad S-wave states are expected as well as some less broad P, D and F states which may be generated in medium and heavy nuclei.

This paper contains three main sections. In section II the η absorption mechanisms are discussed and the two nucleon absorption rate is calculated. Section III is devoted to calculations of the effective $N\eta$ amplitude in the nuclear medium. This is done with a K-matrix approach presented in Ref. [7]. Finally in section IV, some nuclear levels of the η mesons are given.

II. LIFETIME OF THE η MESON IN NUCLEAR MATTER

The η meson lifetime in a nucleus is determined by three basic reactions

$$\eta N \rightarrow \pi N \tag{1}$$

$$\eta (NN)^0 \rightarrow NN \tag{2}$$

$$\eta (NN)^1 \rightarrow NN. \tag{3}$$

Where the superfix denotes the spin of NN pairs. The first process is rather well known. It is described in more detail in the next section as it turns out to be the dominant one . The other two reactions (2) and (3) correspond to η absorption on two correlated NN pairs in either the spin singlet or spin triplet states. The rates for these two-nucleon η capture modes have been uncertain for some time and two extreme views were expressed. In the first papers [1], [2]

these reactions were discarded altogether. On the other hand, a model calculation of Ref. [4] indicated large or even dominant effects from the latter two processes. Now a purely phenomenological evaluation is possible as the cross sections for

$$pp \rightarrow pp\eta \tag{4}$$

$$pn \rightarrow d\eta \tag{5}$$

$$pn \rightarrow pn\eta \tag{6}$$

have been measured in the close to threshold region [9], [10], [6], [11], [12]. The first reaction (4) may be used to calculate the rate of the process (2), where the protons are correlated in a spin singlet state. The second reaction (5) gives the rate of η absorption on a spin triplet NN pair.

General, but approximate, relationships between scattering cross sections and the nuclear decay rates are given by the following formulas for the absorptive optical potentials:

$$W_N(r) = \rho(r)[\frac{1}{2}v_{\eta N}\sigma(N\eta \rightarrow \pi N)] \tag{7}$$

$$W_{NN}^{0,1}(r) = \rho(r)^2[\frac{1}{2}v_{NN}\sigma\left(NN \rightarrow (NN)^{0,1}\eta)\right]\frac{L_1(NN)}{L_2(NN\eta))} \tag{8}$$

$$W_{NN}^{1}(r) = \rho(r)^2[\frac{1}{2}v_{NN}\sigma(pn \rightarrow d\eta)]\frac{L_1(NN)}{L_1(d\eta)\psi_d(0)^2}, \tag{9}$$

where σ are the total cross sections at low energies, v_{NN} is the relative velocity in the NN system required to produce slow a η, $v_{\eta N}$ is the ηN relative velocity, L_1, L_2 are the phase space elements defined by

$$L_1(NN) = \int \frac{d\bar{p}}{(2\pi)^3}\delta[E - E_{NN}(p)]\,,$$
$$L_2(NN\eta) = \int \frac{d\bar{p}d\bar{q}}{(2\pi)^6}\delta[E - E_{NN}(p) - E_\eta(q)]\,,$$

and $\psi_d(0)$ is the deuteron wave function at the origin. These relationships essentially reflect the detailed balance property of the direct and inverse processes. One correction involving the deuteron final state is introduced into the relation (9) with the following motivation. The meson formation reactions require high ($\approx 900MeV/c$) momentum transfer between the two nucleons. Therefore, by the uncertainty principle one expects short distances to be involved and the deuteron to be formed in a coalescent way. Hence it is the deuteron wave function $\psi_d(0)$ that arises in Eq.(9). The experimental cross section $\sigma(pn \rightarrow d\eta) = 93\mu b$ measured in Ref. [11] at the excess energy $Q = 56$ MeV and Eq.(9) generate an absorptive potential of $W_{NN}^1(0) = 4.2$ MeV strength at the nuclear center. On the other hand, the $\sigma(pp \rightarrow pp\eta) = 4.9\mu b$ measured at $Q = 38$ MeV [6] yields $W_{NN}^0(0) = 1.2MeV$, according to Eq.(8). The statistical average $W_{NN} = 3/4W_{NN}^1 + 1/4W_{NN}^0$ obtained in this way is small [$W_{NN}(0) = 3.4$ MeV] and is of less importance than the dominant $W_N(0) \approx 8$ MeV.

Corrections to relation (8) are necessary, since it is based on the detailed balance assumption which has clear limitations in the nuclear medium situation. These corrections are discussed rather schematically as the main effect comes from the deuteron relation (9). Close to the meson production threshold the inverse reactions $NN \rightarrow NN\eta$ rates are strongly enhanced by final state interactions between the two nucleons. These interactions reflect the proximity of the deuteron or the spin singlet virtual state. Such long ranged structures are not formed in the initial states of $\eta NN \rightarrow NN$ reactions inside nuclei. Thus, a correction should be introduced into Eq.(8). It may be expressed in terms of a final state wave function ψ_{NN} in the same way as it was done in Eq.(9) for the deuteron. The usefulness of such a parallel between the bound and scattering states was demonstrated in Ref. [13]. Roughly, the enhancement due to final state interactions is due to large values of ψ_{NN} at short ranges as compared to the values of the incident wave – a j_0 spherical Bessel function. In order to average over the interaction range a simple potential is used to describe reaction (4). The wave function and an integral of a transition matrix element over the three-body phase space are calculated in a way described in Ref. [14]. It follows that an enhancement of the reaction rate due to final state pp interactions amounts to a factor of 5 at $Q = 38$ MeV. The same factor reduces the spin singlet absorptive

potential to an almost negligible value of $W^0_{NN}(0) = 0.2$ MeV. The total nuclear matter average is now $W_{NN}(0) = 3.2$ MeV, and it is this number that is used for further calculations in this paper.

A consistency check for this procedure is provided by reaction (6), which has been studied recently on a deuteron target [12]. For this purpose, the cross section $\sigma(pn \to pn\eta) = 70\mu b$ found at $Q = 56$ MeV is now used. This particular value of Q is chosen to guarantee the final S-wave dominance and to reduce the pn final state interactions at the same time. Reaction (6) involves two pn spin states. Under the assumption of statistical proportion for these states, Eq.(8) generates $W^1_{NN}(0) = 7.7$ MeV. The effect of $pn, S = 1$ final state interactions reduces this estimate by a factor of ≈ 1.5. The final $W^1_{NN}(0)$ compares favourably with the corresponding value obtained from the deuteron.

III. NUCLEAR OPTICAL POTENTIAL FOR THE η MESON

In this section, a scattering amplitude for the η-N system immersed in nuclear matter is studied. This amplitude is based on a phenomenological K-matrix model of Ref. [7]. In the next stage, it is used to calculate the optical potential for η mesons.

Low energy η-N interactions are dominated by two factors: the $S(1540)$ resonance and the cusp at the η-N threshold. The cusp is seen directly in the π-N channel but it has to be calculated in the η-N channel. This scenario is plotted in fig.1, which shows the elastic η-N scattering amplitude. The strength of the cusp reflects the value of the scattering length. On the other hand, as the nucleons are bound it is the region just below the cusp that determines the nuclear optical potential for η mesons. To calculate it we use a model that contains four basic channels: η-N, π-N ,γ-N and $\pi\pi$-N. To simplify the argument it is briefly presented in a single η-N channel case. For numerical calculations this limitation is relaxed, however.

A. The η-N scattering amplitude

To account for the $S(1540)$ resonance the K matrix is parametrized as

$$K = K_B + \frac{ff}{E_0 - E}, \tag{10}$$

where E_0 is the position of the CDD pole related to the resonance and f is its coupling to the η-N channel. The first term K_B describes some potential interactions within the channel. The complete model discussed in Ref. [7] contains in general nine phenomenological parameters, those related to the η-N channel are $f^2 = 0.225$, $E_0 = 1541$ MeV and $K_B = 0.177fm^{-1}$.

Now, the T matrix is obtained from Heitler's equation

$$T = K + iKqT, \tag{11}$$

where q is the relative η-N momentum .

A nuclear optical potential for η may be given in terms of this scattering matrix, if the energy dependence in $T(E_N + E_\eta)$ is accounted for, and an average over nucleon states is taken. To do this a simple Fermi gas is used with a nucleon Fermi level at -8 MeV and nucleon energies given by $E_N = U_N + p_N^2/2M$. The meson energy E_η is put equal to zero, and this simulates the conditions in large nuclei when the η binding energy is close to the bottom of the mesonic optical potential well. The scattering amplitude averaged over the nucleon levels defines an effective scattering length which is denoted by $A_{\eta N}$. The numerical value $A_{\eta N} = 0.50 + i0.092fm$ is obtained and should be compared to the free scattering length $a_{\eta N} = 0.75 + i0.27fm$. The difference between these two quantities reflects the slope of the cusp as well as a rapid fall of the absorptive amplitude in the subthreshold region.

The optical potential is given by $A_{\eta N}$ in the standard way

$$V_N(r) = -\frac{2\pi}{\mu_{\eta N}} A_{\eta N}\rho(r), \tag{12}$$

where ρ is the nuclear density, μ is the η-N reduced mass and the index N on V_N indicates the single nucleon origin of this potential. The absorptive part of V_N given above coincides in the scattering region with the V_N of (7). With $A_{\eta N}$ obtained above from the free scattering amplitudes the optical potential becomes $V_N(0) = -53 - 10i$ MeV. The depth of this well turns out to be much deeper than the pioneering estimates of about -25 MeV obtained in Refs. [1], [2] but the absorptive part is similar .

For the nuclear physics of η mesons, the energies in the η-nucleon system range from about 50 MeV below the threshold to 20 MeV above it. This region is dominated by the $S(1540)$, which is supposed to be determined by some short range (quark) interactions. This inner state is coupled to the channel states which change its properties. In the nuclear medium situation one needs to distinguish these internal and channel states as they are differently affected by the nuclear medium. This question is studied in the next section .

B. Nuclear medium effects

The assumption behind the singularity in our K matrix is that the $S(1540)$ is built upon an internal state N^* (say a quark state) which is additional to the channel states. In this sense it has to appear in any propagators in the intermediate states of interest. The energy of this internal state E_0 contains a contribution S due to its coupling to the channel states. This contribution (or energy shift) may be presented as

$$S = f^2 \int d_3q \frac{(v(q)/v(q_o))^2}{(2\pi)^2 \mu_{\eta N}} \frac{1}{E_{N\eta}(q) - E}, \tag{13}$$

where f is the coupling constant from Eq.(10), $v(q)$ is a formfactor for the $N^*N\eta$ vertex and q_o is an on-shell momentum. While the coupling constant is known in this model the formfactor is not. It has to be very short ranged, in order to describe the physics involved and also to be consistent with Eq.(10), where E_0 is kept constant. It turns out that, for the nuclear physics of interest, the value of S is irrelevant. What is relevant is the change ΔS due to the nuclear medium, and this involves only the low q region in the integral of Eq.(13). The nuclear shift of the $S(1540)$ is thus practically independent of the formfactor. Thus, the main effect of the nuclear medium becomes the Pauli blocking effect which should be introduced into Eq.(13). It pushes the energy of $S(1540)$ upwards. Typical shifts ΔS for the nuclear matter Fermi momentum $1.36 fm^{-1}$ are 36 MeV for a nucleon at the bottom of the well and 19 MeV for a nucleon at the Fermi level. In these calculations the η meson has been put to rest on the bottom of its potential well and the intermediate state meson excitation energies are described by free kinetic energies. Since the energy release in the pionic decay channels is large, the corresponding nuclear effects due to $N\pi$ and $N\pi\pi$ states are insignificant. In principle, the nuclear effects enter also into the term K_B of Eq.(10). However, this term is very small in this model and these corrections are negligible. Readers interested in the details of such calculations are referred to Ref. [15], where similar shifts are discussed in the context of K mesons. In practice, this model of $S(1540)$ and its nuclear behavior are close to the description of the $\Lambda(1405)$ used in Ref. [16], where all the nuclear effects enter via the S of Eq.(13). The nuclear shift ΔS has a surprisingly small effect on the depth of the η meson optical potential well but it reduces the absorption strength. When included into the scattering matrix this shift produces an effective scattering length in the nuclear medium $A_{\eta N} = 0.45 + i0.068 fm$ and the optical potential depth $V_N(0) = -47 - 7.1i$ MeV. This value is used in further calculations.

Another but related point to discuss is an effect of the nuclear absorption on the nuclear properties of the $S(1540)$. The integral in Eq.(13) is normalized in such a way that its imaginary part yields a half width for this state. This width may be changed by a collision damping process i.e. the $N^*N \to NN$ reaction. In the previous section this process was related to the experimental $NN \to NN^* \to NN\eta$ cross section and this relation has generated the two nucleon absorptive potential W_{NN}. One might be tempted to include this process also into Eq.(13) introducing complex energies E and $E_{N\eta}$. Corrections to the optical potential obtained in this way are found to be small both for the real and the absorptive parts. However, as such a method is model dependent these corrections have been dropped altogether.

IV. RESULTS

The basic difference between this and previous calculations is that the η-N model used here produces the η- nuclear optical potential well to be much deeper than those due to other models. The nuclear states of eta mesons are thus bound much more strongly. On the other hand, the absorptive part of this potential is comparable to results obtained elsewhere. The effect of two nucleon capture modes calculated directly from the scattering cross sections is rather moderate. It enlarges the level widths, in particular those of the S states, by about 5 MeV. On the other hand, widths of higher angular momentum states localized at the nuclear surface are less affected, since they involve ρ^2 terms. Some quasibound state energies and half widths in are given in Table I . These were calculated with the nuclear densities following the electric charge profiles. In addition to those states there may arise resonances generated by the centrifugal barrier. For example, a resonance in a G wave is likely to be formed in this way in Pb.

TABLE I. Energies and half-widths of eta states in nuclei, [MeV].

Nucleus	S	P	D	F
^{16}O	–17.2 – i 6.7	–	–	–
^{40}Ar	–27.2 – i 8.6	–11.2 – i 6.6		–
^{208}Pb	–39.2 – i 10.1	–31.5 – i 9.6	–22.6 – i 9.0	–12.3 – i 8.1
^{208}Pb	–19.4 – i 8.6	–8.0 – i 7.3	–	–

The mesons appear to be strongly bound even in light nuclei, in particular the S state obtained in oxygen is bound by about 15 MeV more than the one predicted in Ref. [1].

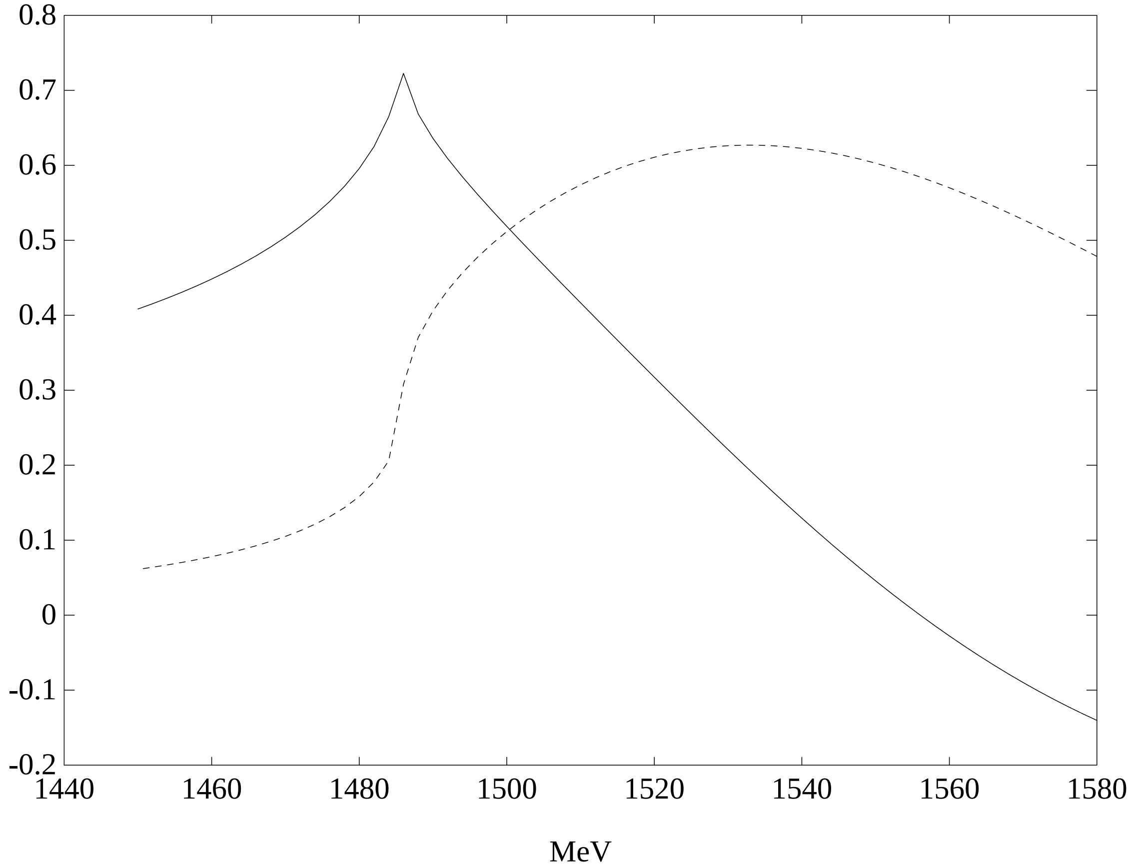
A similar calculation has been performed in very light nuclei. In particular, S wave bound states were obtained in ^{4}He and ^{6}He with the complex energies of $-8.8 - i7.4$ and $-17.2 - i9.5$ MeV. The latter system is of interest in the context of experiments planned at the Brookhaven Laboratory [17], where it may be observed as a final state. The energy given above is based on the structure and radius of ^{6}He found in Ref. [18].

Another effect of the strong attraction is that high angular momentum bound states arise. Some of those upper states may depended on details like: neutron density distributions, the distribution of nuclear momenta in the surface region and the range of ηN forces which have not been discussed here. These effects are of secondary importance and should be discussed only if there is a significant experimental progress in this field. The upper bound states are slightly narrower and in some cases may be separated from the region of strongly overlapping lower states. The experimental search for those states may turn out to be easier.

Acknowledgements. We wish to thank Joanna Stepaniak for many stimulating discussions. S.W. is grateful to the Helsinki Institute of Physics for hospitality and financial support. Support from KBN Grant No 2P0B3 048 12 is also acknowledged.

[1] Q.Haider and L.C.Liu, Phys. Lett. B **172**, 257 (1986); Phys. Rev. C **34**, 1845 (1986).
[2] G.L.Li, W.K.Cheung and T.T.Kuo, Phys. Lett. B **195**, 515 (1987).
[3] B.Chrien et al. , Phys. Rev. Lett. **60**, 2595 (1988).
[4] H.C. Chiang, E.Oset and L.C.Liu , Phys.Rev. C **44** 2595 (1988).
[5] C.Wilkin, Phys. Rev. C **47** ,R938 (1993).
[6] H.Calen et al., Phys.Lett. B**366**, 39 (1996).
[7] A.M.Green and S.Wycech, Phys. Rev. C **55**, R2167 (1997).
[8] M. Batinić, I. Dadić, I. Šlaus, A. Švarc and B.M.K. Nefkens Phys. Rev. C**51**, 2310 (1995); M. Batinić, I. Dadić, I. Šlaus, A. Švarc, B.M.K. Nefkens and T.-S.H. Lee, Physica Scripta **56**, 321 (1997) and Phys. Rev. C **57**, 1004 (1998)
[9] A.M.Bergdold et al., Phys. Rev.D **48** R2969 (1993).
[10] E.Chiavassa et al., Phys.Let. B **322**, 270 (1994)
[11] H.Calen et al., Phys.Rev.Lett. **79**, 2642 (1997)
[12] T.Johansson for Promice-WASA Collaboration, TSL/ISV-97-183
[13] G. Fäldt and C. Wilkin, Nucl. Phys. A **604**, 441 (1996).
[14] S.Wycech, Acta. Phys. Pol.B **27**, 2981 (1996).
[15] R. Staronski and S.Wycech, J.Phys. G **52**, 544 (1995).
[16] J.M.Eisenberg, Phys.Rev. C **14**, 2343 (1976)
[17] B. Nefkens, private communication.
[18] G.D.Alkhazov et al., Phys.Rev.Lett. **78**, 2313 (1997)

FIG. 1. The elastic η-N scattering amplitude in $fermi$ units plotted against the C.M. energy Ecm in MeV. Real part - continuous line, absorptive part - dashed line.



0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
1440
1460
1480
1500
1520
1540
1560
1580
MeV